PYTHAGORAS MUSICUS

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The legacy of Pythagoreanism pervades Western thought. After flourishing as a school from the sixth to the fourth centuries B.C., Pythagoreanism eventually merged with Platonism. In its Platonized form it was revived in the time of Cicero, transmitted to the Middle Ages via Boethius' quadrivium,[1] and revived once again during the Renaissance.[2] A further recrudescence may well be upon us as the discoveries of modern physics are found to be remarkably anticipated by Pythagorean notions of the mathematical quantification of nature.[3]

Of all Pythagoras' reputed achievements the most perisitent and

1) Boethius coined the term in his introduction to *De Institutione arithmetica*, but Iamblichus was the first to denote arithmetic, music, geometry, and spherics as the "four steps in set order to ascend to wisdom," a concept which may have originated with Archytas. See Calvin Bower, "Boethius and Nicomachus: An Essay Concerning the Sources of *De Institutione musica*," *Vivarium* 16(1978), 44 and Charles André Barbera, "*Republic* 530C—531C: Another Look at Plato and the Pythagoreans," *American Journal of Philology* 102(1981), 402—403. For a translation and discussion of the pertinent passage from Archytas, see Alan Bowen, "The Foundations of Early Pythagorean Harmonic Science: Archytas Fragment 1," *Ancient Philosophy* 2(1982), 82, 85—86.

2) See S.K. Heninger, *Touches of Sweet Harmony: Pythagorean Cosmology and Renaissance Poetics*(San Marino, Calif.: Huntington Library, 1974).

3) Alfred North Whitehead, *Science and the Modern World*(Toronto: Macmillan, 1925), 27—37 and Samuel Sambursky, *The Physical World of the Greeks*(London: Routledge & Kegan Paul, 1956), 45—48.

richly attested are those in music. He, above all, "invented" music by determining the ratios of the musical concords. This felicitous event is said to have occurred by chance one day as Pythagoras passed a blacksmith shop. Nicomachus of Gerasa in his *Harmonikon enchiridion* of the mid-second century A.D. provides the first extant version of the tale:

> One day he [Pythagoras] was deep in thought and seriously considering whether it could be possible to devise some kind of instrumental aid for the ears which would be firm and unerring, such as sight obtains through the compass and the ruler or the surveyor's instrument; or touch obtains with the balance or the device of measures. While thus engaged, he walked by a smithy and, by divine chance, heard the hammers beating out iron on the anvil and mixedly giving off sounds which were most harmonious with one another, except for one combination. He recognized in these sounds the consonance of the octave, the fifth and the fourth. But he perceived that the interval between the fourth and the fifth was dissonant in itself but was otherwise complementary to the greater of these two consonances. Delighted, therefore, since it was as if his purpose was being divinely accomplished, he ran into the smithy and found by various experiments that the difference of sound arose from the weight of the hammers, but not from the force of the blows, nor from the shapes of the hammers, nor from the alteration of the iron being forged. Having carefully examined the weights of the hammers and the impacts, which were identical, he departed to his home.[4]

Nicomachus goes on to relate how Pythagoras confirmed his

4) Flora R. Levin, "Nicomachus of Gerasa Manual of Harmonics: Translation and Commentary," (unpublished Ph.D. dissertaton, Columbia University, 1967), 28—29.

discovery by pluckin ɛ on strings with attached weights equal to those of the four hammers(viz. 6, 8, 9, 12) suspended from a stake fixed diagonally to the walls of his house. He then transferred the idea of weighted strings to strings under analogous amounts of tension on a "string stretcher"(chordotonon), and finally extended the test to "the striking of plates, to auloi and panpipes, to monochords and triangular harps, and the like."[5]

Nicomachus could not have tried these experiments himself; had he done so he would have encountered unsettling results with several of them. Hammers of various weights beating on the same anvil, for example, produce sounds differing in loudness but not pitch—pitch in this case being decided by ths size of the anvil rather than by the weight of the hammers. With weighted strings, moreover, pitch is proportional not directly, but to the square root of the appiled tension. To yield an octave, a given weight on a string would thus have to be quadrupled rather than doubled, e.g., $2 : 1 = \sqrt{24} : \sqrt{6}$. Although the acoustical inaccuracies in Pythagoras' discovery were already noted by Nicomachus' younger contemporary, Claudius Ptolemy,[6] it was not until the early seventeenth century that the acoustical properties of weighted strings were scientifically examined.[7]

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5) *Ibid.*, 31.

6) Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. E. L. Minar(Cambridge, Mass.: Harvard University, 1972), 376 and Flora Levin, *The Harmonics of Nicomachus and the Pythagorean Tradition* (University Park, Pa.: The American Philological Association, 1975), 69.

7) Marin Mersenne, *Questions harmoniques*(Paris, 1634), 166. Mersenne was stimulated in the study of sound by Galileo, who was the first to point out the acoustical errors of Pythagoras' experiments in his *Discorsi e dimostrazione matematiche.* See Sir John Hawkins, *A General History of the Science and Practice of Music*(London, 1776), I (New York: Dover, 1963), 10—12. The role of experiment in modern science appears to have arisen from the tuning controversies between Zarlino and Vincenzo

Figure 1. Woodcut from Franchino Gaffurio's *Theorica musicae* of 1492 depicting the discovery of music.

Galilei in the late sixteenth century. The notes of Vincenzo's acoustical experiments passed to his son who incorporated them into his later work. See Stillman Drake, *Galileo Studies*(Ann Arbor: University of Michigan, 1970), 43—62.

For sixteen hundred years after Nicomachus the legend of the "harmonious blacksmith" evolved through a series of religious, musical, and historical treatises. The famous woodcut from Franchino Gaffurio's *Theorica musicae* of 1492, for instance, portrays in microcosm much of what was believed about Pythagoras as musician in the Middle Ages and Renaissance(see Figure 1). From this it is evident, however, that by the late fifteenth century Nicomachus' original story had acquired numerous accretions, especially in the number of hammers and associated personae. This paper will trace the origin of these accumulations and then investigate the ancient sources of the legend itself.

I

The first to embellish the details of the Pythagorean musical discoveries beyond Nicomachus was Boethius in his *De Musica* of the sixth century A.D. Before him, in the late third century, Iamblichus had recounted the tale in his biography of Pythagoras but copied word from Nicomachus; a century later Macrobius told the story most vividly in his commentary on Cicero's *Somnium scipionis* but contributed nothing new save the quaint expression (following Pythagoras' request for the smithies to exchange hammers), "the difference in tones did not stay with the men but followed the hammers."[8] This phrase was incorporated into many later retellings, including that of Boethius, who interjected a fifth hammer and the experiment with water-filled glasses into the legend:

And since perchance there were five hammers, one was found

8) Macrobius, *Commentary on the Dream of Scipio*, trans. W.H. Stahl(New York: Columbia University, 1952), 186.

to weigh twice as much as another, and these two resounded a diapason consonance. The one which had weighed twice as much as a second formed the sesquitertian relation of a third with which naturally it produced a diatessaron. He found the one which weighed twice as much as a second to be the sesquialter relation of a fourth, which was related to it by a diapente consonance. Those two, to which the above one of double weight was proved to be sesquitertian and sesquialter relation, were discovered in turn to be related by the sesquioctave proportion. The fifth hammer, which was dissonant with all, was rejected. ...also, following this way of measuring, he placed measured weights of water in glasses, and then struck these glasses prepared with various weigts with a small twig or copper rod; and he rejoiced to find nothing in conflict.[9]

Although no more workable, the water glass experiment was simply an extension to a different medium of the earlier experiment with weighted strings. The interpolation of an additional hammer only to be rejected was apparently made for dramatic purposes alone; but, whatever the reason may have been, music theorists thereafter subscribed to either the tradition of four hammers stemming from Nicomachus(e.g., Aurelian, Marchettus of Padua, Tinctoris) or five hammers stemming from Boethius(e.g., Guido, John of Afflighem, Johannes de Muris, Adam of Fulda, Zamorensis) until Gaffurio himself proposed a sixth.[10]

For the first part of his account Gaffurio followed Boethius by mentioning five hammers and eliminating one as discordant. He

9) Calvin Bower, "Boethius' *The Principles of Music*, An Introduction, Translation, and Commentary(unpublished Ph. D. dissertation, George Peabody College for Teachers, 1967), 60—63. For Boethius' knowledge of acoustics see Jean Edmiston, "Boethius on Pythagorean Music," *Music Review* 35(1974), 183—184. See also note 79.

10) For a detailed history see Hans Oppermann, "Eine Pythagoraslegende,"

then suggested that more hammers must nevertheless have been present in order to include all the notes of the octave:

> During this daily investigation of the secrets of harmony, Pythagoras also explored(I am persuaded) not only the combinations of these four tones, but also the other chords by which the art of music is perfected and nurtured. Primarily there are six combinations of tones... Hence we believe that Pythagoras exchanged the fifth, inharmonious hammer for another, and then added a sixth... Now the order of the six hammers according to weight would have been as follows—4, 6, 9, 12, 16.[11]

To explain the presence of Jubal and Philolaus as companions to Pythagoras in the woodcut would have been unnecessary to educated Medieval and Renaissance musicians. Philolaus, a Pythagorean and contemporary of Socrates, was born in Magna Graecia(in either Croton or Metapontum depending on the authority) but settled in Thebes after the political uprisings against the Pythagoreans in the middle of the fifth century B.C.[12] There he collected about him a body of disciples and produced the first written record of Pythagoreanism. The extant fragments of this books, although their authenticity is disputed by some, reveal Philolaus to have had musical, mathematical, and cosmological interests.[13]

Bonner Jahrbücher 130(1925), 284—301.

11) Quoted in John M. Steadman, "The 'Inharmonious Blacksmith': Spenser and the Pythagoras Legend," *Proceedings of the Modern Language Association* 79(1964), 665. See also Oppermann, 289—290, 295.

12) Philolaus' *floruit* is established by a reference in Plato's *Phaedo* 61e. Very little about his life is known. For his biography and teachings see Kathleen Freeman, *The Pre-Socratic Philosophers*(Oxford: Alden Books, 1946), 220—232 and W.K.C. Guthrie, *A History of Greek Philosophy* I (London: Cambridge University, 1962), 329—333. For Pythagorean politics see below.

13) For an English translation of the fragments see Kathleen Freeman, *Ancilla*

Jubal, of course, is Pythagoras' Biblical counterpart. For his connection with blacksmithing we must turn to Isidore of Seville, who in his *Etymologies* of the seventh century A.D. was the first to place the Christian tradition side by side with the pagan:

> Moses says that the inventor of the art of music was Tubal, who was of the race of Cain, before the flood. The Greeks say that Pythagoras found its beginnings in the sound of hammers and the striking of stretched strings.[14]

The substitution of "Tubal" for "Jubal," which appears in most manuscripts of the *Etymologies*, resulted from an orthographical mistake made by copyists of early Vulgate manuscripts. The Medieval practice of not capitalizing proper names, thereby obscuring the difference between "i"(which also served as the Latin "j") and

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to the Pre-Socratic Philosophers(Cambridge, Mass.: Harvard University, 1962), 73—77. The literature on the controversy over their authenticity is vast, extending from their first publication in 1819 by Boeckh to the present. Surveys of the lengthy dispute may be found in G.S. Kirk and J.E. Raven, *The Presocratic Philosophers*(Cambridge: University Press, 1957), 308—318 and Holger Thesleff, *An Introduction to the Pythagorean Writings of the Hellenistic Period*(Abo: Abo Akademi, 1961), 41—45. The general scepticism expressed by Ingram Bywater("On the Fragments Attributed to Philolaus the Pythagorean." *Journal of Philology* [1868], 21—53), John Burnet(*Early Greek Philosophy*, 4th ed. [New York: World Publishing, 1957], 277—284), and Erich Frank(*Plato und die sogenannten Pythagoreer* [Halle: Niemeyer, 1923]) has given way to at least partial acceptance of the fragments as geniune. See criticism of Bywater's arguments in Guthrie, *History*, 329—333 and of Frank's in George de Santillana and Walter Pitts, "Philolaus in Limbo, *or:* What Happened to the Pythagoreans?" *Isis* 42(1951), 116—120. Burkert, *Lore and Science*, 218—277 provides the most exhaustive recent examination of the fragments which he pronounces almost wholly authentic. See commentary on Burkert's analysis in J.A. Philip, *Pythagoras and Early Pythagoreanism* (Toronto: University Press, 1966), 119—122. For Philolaus' musical accomplishments see below.

14) Quoted in Oliver Strunk, *Source Readings in Music History*(New York: W.W. Norton, 1950), 94.

"t," and the proximate occurrance of "Tubalcain" in Genesis 4: 19—22 were undoubtedly responsible for the problem:[15]

> And Lamech took unto him two wives: the name of the one was Ada, and the name of the other Zillah. And Ada bare Jable: he was the father of such as dwell in tents, and of such as have cattle. And his brother's name was Jubal: he was the father of all such as handle the harp and organ. And Zillah she also bare Tubalcain, an instructor of every artificer in brass and iron.

"Tubal," meaining "Jubal," is therefore often seen in Medieval treatises. On the other hand, the historian Flavius Josephus of the first century A.D., who was cited as an authority by many later writers, employed "Tubal" for "Tubalcain" in his amplification of the Biblical narrative in *The Antiquities of the Jews.*[16] Despite this opportunity for confusion, music theorists of the Middle Ages never mistook Jubal/Tubal the musician for Tubal/Tubalcain the smith, although this error has been made by modern art historians in their interpretations of Medieval Bible illustrations or of stained glass windows depicting the invention of music.[17]

The fusion of Jubal with smithing was made in Peter Comestor's *Historica scholastica* of c. 1170, a required textbook at what was

15) Paul E. Beichner, *The Medieval Representative of Music, Jubal or Tubalcain?*(South Bend, Ind.: University of Notre Dame Medieval Institute, 1954), 8. On the relationship of Jubal and Pythagoras see also Judith Cohen, "Jubal and the Middle Ages," *Yuval: Studies of the Jewish Research Centre* Ⅲ, ed. Isreal Adler and Bathja Bayer(Jerusalem, 1974), 83—99 and James W. McKinnon, "Jubal vel Pythagoras quis sit inventor musicae?" *Musical Quarterly* 64(1978), 1—28.

16) Beichner, 9. This is not true of all Josephus manuscripts; see, for example, *Jewish Antiquities* Ⅰ, 31, where Tubalcain is called "Jub êl".

17) Beichner, 18—27.

later to become the University of Paris. The book by the "Master of Histories," as Comestor was called, was enormously influential for centuries, studied even more than the Bible itself. It was a universal history consisting largely of a *mélange* of Biblical and mythological lore with commentary synthesized from a variety of Christian, Jewish, and pagan sources.[18] After ascribing the invention of music to Tubal(Jubal), Comestor continues:

> 'And Zillah she also bare Tubalcain,' who was the first inventor of the art of iron. He skillfully exercised the art of war and made sculptures out of metalwork for the delight of the eyes. While he was working, Tubal, whom we have mentioned above, delighted in the sound of metals and devised out of their weights the proportions and consonances which originated in them. This invention has erroneously been attributed by the Greeks to Pythagoras.[19]

Chronology was a preoccupation of Comestor and other twelfth-century Biblical scholars. Because they were persuaded of Jubal's chronological priority over Pythagoras, the former became music's preferred inventor in subsequent writings.[20] Cerone's *El Melopeo* of 1613 provides a late example:

> The manner in which Jubal discovered the proportions [of musical harmony], was as follows. One day, upon entering the forge of his brother Tubalcain, the inventor of the blacksmith's art, Jubal heard the hammers produce a harmonious concord, because one was heavy, another light, and [the others] of

18) Cohen, 84. See also Saralyn R. Daly, "Peter Comestor: Master of Histories," *Speculum* 32(1957), 62—73.
19) Quoted in Cohen, 85.
20) McKinnon, 9.

medium weight. Being naturally inclined to music and delighting in the sound of the hammers, which gave such sonorous strokes, Jubal discarded the fifth hammer and weighed the four remaining, in order to learn what proportions produced this concord ...[21]

In the seventeenth and eighteenth centuries the story of the invention of music got taken up by the early music historians—Calvisius, Kircher, Printz, Martini, Gerbert, Hawkins—in works often as euhemeristic as their Medieval forerunners but even more encyclopedic and increasingly empirical in approach. The gradual incorporation of the acoustical fallacies identified by Galileo and Mersenne into the account and the growing scepticism of the Enlightenment restored the historical Pythagoras to his former musical prominence over the quasi-mythological Jubal. The historians' search for music's inventors finally concluded upon the emergence of the Romantic conception of music originating in nature or the human heart.[22]

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If the apocryphal story of Pythagoras in the blacksmith shop began with Nicomachus what are its roots? It appears that a connection between Pythagoras and the origin of music theory goes back at least to Plato's time. Porphyry reports in his commentary on Ptolemy's *Harmonica:*

Heraclides in his *Intorduction to Music* writes as follows:

21) Quoted in Steadman, 665.
22) McKinnon, 18—23.

> Pythagoras, so Xenocrates says, discovered that the musical intervals also owe their origin of necessity to number, because they consist in a comparison of one quantity with another. He further investigated in what circumstances the intervals are concordant or discordant, and in general the origin of all harmony and disharmony.[23]

As Guthrie observes, the above passage resembles a "Chinese box arrangement" of quotation within quotation. Porphyry, in the third century A.D. quotes Heraclides Ponticus from the fourth century B.C., who in turn had quoted Xenocrates, his contemporary and fellow Platonic disciple. No earlier attribution exists so one can only hypothesize on Pythagoras' actual musical accomplishments.

Who, then, was Pythagoras and what did he teach? What little is known of Pythagoras' life consists of an uncertain mixture of fact and legend. Even his name, "the man by means of whom the Pythia is talking," implies *daimon* status, for he seems to have been mythologized already during his lifetime.[24] Most ancient sources agree, however, that Pythagoras was an Ionian, born in Samos of a gem engraver, Mnesarchos, and flourished in the Sixtieth

23) Quoted in Guthrie, *History*, 222.

24) The classical *Vitae Pythagorae* of Diogenes Laertius, Porphyry, and Iamblichus date from the third to fourth century B.C. For discussions of their mutual relationships and sources see J.A. Philip, "The Biographical Tradition—Pythagoras," *Transactions of the American Philological Society* 90(1959), 185—194 and Burkert, *Lore and Science*, 97—109. As a group the *Vitae* are characterized by their "love for the marvelous" and "singular lack of any critical faculty" in their compilation(Guthrie, *History*, 156). The tradition concerning Pythagoras' birthplace, paternity, family, education, voyages of instruction, writings, and death are conveniently collated in Philip, *Pythagoras*, 185—199. Diogenes' biography forms Chapter Ⅷ of his *Lives of Eminent Philosophers* Ⅱ; Porphyry's is translated in Moses Hadas and Morton Smith *Heroes and Gods: Spiritual Biographies in Antiquity*(New York: Harper & Row, 1965), 105—128; and that of Iamblichus is translated by Thomas Taylor(London, 1818; repr. London: John M. Watkins, 1965).

Olympiad(540—536 B.C.), which suggests an early sixth-century birthdate.[25] As a youth he studied with Pherecydes of Syros, said to be the first Greek to write prose, and as a young man traveled to Egypt, Babylonia, and other Eastern kingdons where he absorbed their ancient and esoteric wisdom.[26] Renowned as a polymath Pythagoras left Samos at the age of forty because of the growing tyranny of its despot, Polycrates.[27]

For unknown reasons Pythagoras chose to emigrate to Croton in Magna Graecia, the large Greek colony in southern Italy, where he founded his celebrated Society.[28] His arrival there roughly coincided with Croton's disastrous defeat by the Locrians at the river Sagras, c. 540 B.C.[29] Details of Society organization and daily life are uncertain. Plato writes simply that Pythagoras instituted a "way of life,"[30] while Plato's contemporary, Timaeus, the great Sicilian

25) The most detailed investigation of Pythagorean chronology is by C.J. De Vogel, *Pythagoras and Early Pythagoreanism*(Assen: Van Gorcum, 1966), 20—27. The author summarizes earlier speculations and proposes the following "corrected scheme": birth, c. 570 B.C.; years of study, 558—554; stay in Egypt and Babylon, 554—533; Samos and journeys to Sparta and Crete, 533—529; Croton, 529—509; death in Metapontum, c. 509.

26) Nancy Demand, "Pythagoras, Son of Mnesarchos," *Phronesis* 18(1973), 91—96, suggests a practical reason for these foreign excursions—family business trips. For another theory see following note.

27) This report, the cornerstone of Pythagorean chronology, is attributed to Aristoxenus by Porphyry(Hadas and Smith trans., 110). According to Herodotus, *Histories* ii. 53, Polycrates said "he would get more thanks if he gave a friend back what he had taken than if he never took it at all." His reign was from c. 532—522 B.C. but the "tyranny" was probably begun by his father Aiakes in c. 560, which may also have prompted Pythagoras' extended visits abroad. See Mary White, "The Duration of the Samian Tyranny," *Journal of Hellenic Studies* 74(1954), 36—43.

28) He could have been influenced in his choice by Democedes of Croton, court physician to Polycrates(Guthrie, *History*, 174) or attracted to the missionary possibilities in a city severely shaken by war(De Vogel, 58).

29) A proverbial battle in which a reputed force of 130,000 Crotonians were routed by 15,000 Locrians and Rhegians. For a full description see Thomas Dunabin, *The Western Greeks*(Oxford: Clarendon Press, 1948), 358—360.

historian, describes a communistic fraternity divided hierarchically into novices(*exoterikoi*) and adepts(*esoterikoi*), modeled loosely upon the *hetaireia* or Greek institution of military-political clubs.[31]

The Society came to wield considerable political influence in Croton by either ruling directly as a group or, as is more likely, by achieving its aims through the concerted lobbying of individual members.[32] Particularly after Croton's defeat of its rival Sybaris

30) *Republic* 600b. This is Plato's only mention of Pythagoras by name in his extant writings. Herodotus ii. 81, in a problematical passage, is the first to mention the existance of a Pythagorean sect: "In this they follow the same rule as the ritual called Orphic and Bacchic, but which is in truth Egyptian and Pythagorean; for neither may those initiated into these rites be buried in woolen wrappings." See further Guthrie's valuable commentary(*History*, 159—160).

31) In Porphyry(Hadas and Smith trans., 36—37), novices are the *acusmatici* who hear only the unexplained, summarized instructions of Pythagoras, while the adepts are the *mathematici* who are favored with his teachings in depth and detail. Iamblichus(Taylor trans., 42—43), on the authority of Timaeus, speaks also of *acusmatici* and *mathematici* whom he equates respectively with "Pythagorists"(*Pythagoristai*) and "Phthagoreans"(*Pythagoreioi*). The latter are the "genuine" followers of Pythagoras, living communally with the Master, but the former are merely "emulators of these," meeting regularly with the others although retaining their own possessions and living apart. The money and property of the commune are managed by *Politikoi*(Taylor trans., 38). For an exhaustive examination of the Timaean evidence see Edwin L. Minar, "Pythagorean Communism," *Transactions of the American Philological Association* 75 (1944), 34—46. Minar concludes that there appear to have been three grades of membership within the Order—initiates serving a three-year probation, apprentices(*acusmatici?*) undergoing a five-year vow of silence, and full members(*mathematici?*). See Burkert, *Lore and Science*, 192—208, for a thorough review of the ancient sources on the *acusmatici* and *mathematici* as well as the possibility of their constituting separate religious and scientific movements within later Pythagoreanism. Of modern authorities, Philip(*Pythagoras*, 138—141) is the most sceptical about the existance of a highly organized and communistic brotherhood; such an institution as described by Iamblichus, he says, would be "unique in the Greek world before the Christian era." On the *hetaireia* see Edwin Minar, *Early Pythagorean Politics*(Baltimore: Waverly Press, 1942), 19—28 and J.S. Morrison, "Pythagoras of Samos,; *Classical Quarterly* n.s. 6(1956), 150—152.

32) "No outstanding thinker in the small society of a city-state could avoid

in 510 B.C.[33] and her increased hegemony in southern Italy as a result, Pythagorean political activity spread throughout the region.[34] The division of the spoils of the Sybarite War ignited a brief revolt against the Pythagoreans in their home city, followed by a widespread bloody and decisive revolution against them fifty years later. After an ensuing diaspora, Pythagoreans were not allowed to return to their homes from abroad for many years until a truce was finally effected by the Athenians.[35] Pythagoras had died in the meantime shortly after escaping with some friends to Metapontum following the Crotonian uprising.[36]

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Pythagoreanism, inasmuch as it can be traced back to the Master himself, has puzzled modern scholars. On the one hand stands Pythagoras the hierophant, shaman, or *acusmaticus* admonishing

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playing some part in public affairs"(Dunabin, 361). For studies of Pythagorean politics see, (in addition to the Minar study cited in note 31), Kurt von Fritz, *Pythagorean Politics in Southern Italy*(New York: Columbia University, 1940). Von Fritz(p. 96) presumes that "Pythagorean rule" may have been analogous to the method of political influence exerted by the Freemasons in the eighteenth century.

33) Herodotus v. 44—45; vi. 21. On reasons for the start of the war and an account of the battle see Dunabin, 362—364. Music is reported to have played a part in Sybaris' defeat. According to Athenaeus(*The Deipnosophists* xii. 520c), the Sybarites taught their horses to dance to the sound of the flute. Croton thus dressed flute players in military uniforms and had them play to the horses, causing them to dance and their riders to be captured.

34) Evidence of Croton's hegemony is based largely on the interpretation of alliance coins. See Dunabin, 355—356 and Minar, *Pythagorean Politics*, 36—37. Guthrie(*History*, 176—177) has Pythagoras responsible for inventing the special incuse coinage. See further discussion by De Vogel, 50—56 and Philip, *Pythagoras*, 197—199.

35) Croton was an Achaian colony(Dunabin, 356).

36) For the traditions concerning Pythagoras' death see Philip, *Pythagoras*, 191—192.

against the eating of beans[37] or preaching the transmigration of souls. On the other hand stands Pythagoras the scientist, philosopher, or *mathematicus* perfecting geometry or establishing the foundations of mathematics. The two sides of his character are undeniable but seem incompatible—in the words of Bertrand Russell, Pythagoras is a curious blend of Mary Baker Eddy and Albert Einstein.[38]

Knowledge of the historical Pythagoras is greatly impeded by his having written nothing,[39] his Society's maintenance of an

37) The curious Pythagorean taboo against beans may have originated for medical reasons. See Robert S. Brumbaugh and Jessica Schwartz, "Pythagoras and Beans: A Medical Explanation," *Classical World* 73(1980), 421—422. The traditional reasons for abstaining from beans are listed by Diogenes Laertius(viii. 34) on the authority of Aristotle. See also Porphyry, Hadas and Smith trans., 121—122. Pythagoras' death, according to one story, resulted from his refusal to enter a beanfield when pursued by enemies (Iamblichus, Taylor trans., 100—101).

38) *A History of Western Philosophy*(New York: Simon & Schuster, 1945), 29. Modern Pythagorean scholarship is surveyed in Burkert, *Lore and Science*, 1—14. Virtually all the primary source materials have been known since the mid-nineteenth century so disagreement among authorities is largely the result of differing interpretations. This situation is reflected quite dramatically by the four most recent and most comprehensive studies of Pythagoreanism by Guthrie(cited in note 12), Burkert(cited in note 6), De Vogel(cited in note 25), and Philip(cited in note 13). All four authors meticulously examine the nature of pre-Platonic Pythagoreanism in an attempt to determine the Society's original teachings and beliefs. Their conclusions vary considerably. Burkert finds early Pythagoreanism exclusively shamanistic; the scientific achievements recorded by Aristotle he regards as late developments because Aristotle refers only to "the Pythagoreans" or to the "so-called Pythagoreans" and not to Pythagoras himself(*Lore and Science*, 29—30, 216). Guthrie(*History*, 155), De Vogel (pp. 1—2), and Philip(*Pythagoras*, 34—35, 69—70, 173), see Aristotle's avoidance of personal names as characteristic of his literary style and assign to Pythagoras and his immediate disciples various advances in mathematics, astronomy, and music. Philip restricts himself to Aristotle's testimony while Guthrie and De Vogel are more liberal in their acceptance of later sources. Guthrie personifies Pythagoras as a great scientist and De Vogel sees him primarily as a great moral teacher.

39) The list of works by Pythagoras in Diogenes Laertius viii. 6 is apocryphal.

unusual silence about its activities,[40] and the Pythagoreans' practice of *autos efa*("he said it") in which all doctrine was attributed to Pythagoras himself.[41] Add to these problems the lack of any substantial testimony of Pythagoreanism before the fourth century B.C. and the Platonization of Pythagoreanism with its subsequent revivals mentioned above, which comprise in all a millenium of accumulated and expanding tradition; it becomes difficult if not impossible to clearly distinguish Pythagoras' own teaching from that of his successors.

Pythagorean scholarship is thus forced to rely heavily on source criticism and circumstantial evidence in its research, leading to widely divergent opinions and continuous debate. Were it not for Aristotle's numerous references to presocratic philosophy in his various works and his distinction between earlier and later thought, attempts to reconstruct Pythagoreanism before Plato would be futile indeed.[42] A comprehensive summary of early Pythagorean religious beliefs and scientific achievements along with their

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See a review of the ancient evidence in Burkert, *Lore and Science*, 218—220.

40) Porphyry, Hadas and Smith trans., 113. The Pythagoreans developed a cult of secrecy. See Iamblichus, Taylor trans., 36, 51, 87, 99, 102, 105.

41) Iamblichus, Taylor trans., 103.

42) Aristotle uses earlier philosophy mainly as a foil for his own theories. He nevertheless "shows a stronger historical sense than most original philosophers possess and had at least the advantage over us in that he was an Ionian Greek like the men of whom he was writing and that he was judging them on fuller evidence than we are"(W.K.C. Guthrie, "Aristotle as Historian," in *Studies of Presocratic Philosophy* I, ed. D.J. Furley and R.E. Allen [New York: Humanities Press, 1970], 243—244). See also J.G. Stevenson, "Aristotle as Historian of Philosophy," *Journal of Hellenic Studies* 94(1974), 139—143. Aristotle's lost essay on the Pythagoreans is also the source of the *acusma* and *mirabilia* tradition. See J.A. Philip, "Aristotle's Monograph 'On the Pythagoreans,'" *Transactions of the American Philological Association* 94(1963), 185—198 and the English translation of the fragments in *The Works of Aristotle* XII, trans. W.D. Ross(Oxford: Clarendon Press, 1952), 134—146.

attendant controversies would be too extensive to undertake here and is, at most, tangential to our present topic. I will confine myself instead to the most famous and most characteristic Pythagorean dogma, *mundum regunt numeri*, as revealed in Aristotle's *Metaphysics*, where first mention of it is made.

For Aristotle the concept of numbers as the basic "stuff"(*physis*) or first principles(*archai*) of the universe was imcomprehensible:

> When they [the Pythagoreans] construct physical bodies out of number—things that possess lightness and weight out of elements which possess neither—they appear to be talking about some other universe and other bodies, not those that we perceive.[43]

To the Pythagoreans, therefore, numbers had magnitude and in some metaphysical way constituted(the Pythagoreans said "were") the world.[44] This new emphasis on form distinguished the Italian from the more materialistic Ionian or Melesian presocratics, who had earlier declared the essence of the univers to be water, air, or the void.[45] The reasoning for the procedure of numbers becoming things as abstracted from Aristotle's criticism could be reconstructed as follows: The ultimate elements, Limit(*peras*) and Unlimited (*apeiron*), embody the respective but secondary notions of Even and Odd, which generate the One(Unit), which, in turn, generates numbers which form things. Causal relationships between successive levels are somewhat vague, but Aristotle explains the correspondence of Limit to Odd and Unlimited to Even as resembling the result of

43) *Metaphysics* 1090a30.
44) This synopsis of "numbers as things" is based on Guthrie's lucid discussion in *History*, 229—251.
45) E.g., Thales, Anaximenes, and Anaximander respectively.

a carpenter's square or gnomon applied to numbers represented geometrically as dots(psephoi-arithmetic). When gnomons are placed around odd numbers, the shape is "limited" by remaining the same, but when placed around even numbers, the shape varies infinitely or "limitlessly"(see Figure 2).[46]

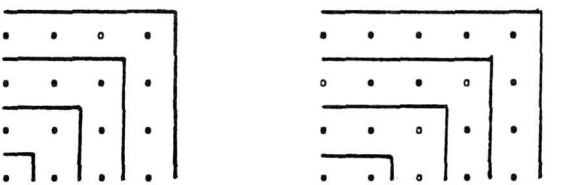

Figure 2. Aristotle's demonstration of Odd as Limit and Even as Unlimited by the application of gnomons.

An Orphico-Pythagorean cosmogony[47] preserved by Aristotle[48] describes the origin of the universe in yet more physical terms:

46) *The Physics* 203a4: "They say moreover that the unlimited is the even, for this when it is enclosed and limited by the odd provides the unlimited element in existing things. This is illustrated by what happens when gnomons are placed around numbers: when they are placed round the one, and without the one, in the one case the figure produced varies continually, whereas in the other it is always the same." For interpretations see F.M. Cornford, *Plato and Parmenides*(London: Kegan Paul, 1939), 6—10 and Theo G. Sinnige, "Pythagoras and Pythagoreanism," in *Matter and Infinity in the Presocratic Schools and Plato*, 2nd ed.(Assen: Van Gorcum, 1971), 70—71. On psephoi or "pebble" arithmetic as a characteristically Pythagorean method of calculation see Burkert, *Lore and Science*, 427—436.

47) On orphism see Martin Nilsson, "Early Orphism and Kindred Religious Movements," *Harvard Theological Review* 28(1935), 181—230 and W.K.C. Guthrie, *Orpheus and Greek Religion*(New York: W.W. Norton, 1966). A relationship between Orphism and Pythagoreanism is implied as early as Herodotus(see note 30), but how much doctrine the two sects shared and the amount of mutual influence is not definitely known. The groups seem to have had similar teachings concerning the soul(see note 65), ritual purity, as well as the origin of the universe. Magna Graecia was a center for Orphic beliefs and Pythagoras was said by late writers(e.g., Ion of Chios) to have authored Orphic verses. See also Minar, *Pythagorean Politics*, 125—132 and Sinnige, 49—63.

48) *The Physics* 213b22.

The promordial "Unit-Sperm" impregnates the Unlimited and begins to inhale it, a process regarded as imposing Limit on the Unlimited. Through repeated breathing the universe becomes discontinuous and discrete bodies emerge. *Harmonia*[49] reigns over all, creating out of chaos a *kosmos*[50] emitting "music of the spheres."[51]

Ⅳ

In a field replete with so much controversy, nothing is more agreed upon by modern authorities than the connection between Pythagorean music theory and the doctrine of numbers as things. Such a connection seems supported by Aristotle:

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49) For a study of the term see Thomas Mathiesen, "Problems in Terminology in Ancient Greek Theory: 'APMONÍA'," in *Festival Essays for Pauline Alderman.* ed. B.L. Kasson(Provo, Utah: Brigham Young University, 1976), 3—17. See other helpful discussions in Flora R. Levin, "The Hendecachord of Ion of Chios," *Transactions of the American Philological Association* 92(1961), 299—300 and Warren Anderson, *Ethos and Education in Greek Music*(Cambridge, Mass.: Harvard University, 1966), 37, 191—195. De Vogel(p.219) argues for Pythagoras as the inventor of the term.

50) The Greek concept of *kosmos* is discussed most extensively in Guthrie, *History,* 208—212.

51) Plato, after affirming the sororal connection made by the Pythagoreans between music and astronomy(*Republic* 530d), provides(the first exposition of cosmic music in the "Myth of Er"(*Republic* 617b). Aristotle(*Metaphysics* 1986a2—3) relates how the Pythagoreans, in supposing the elements of number to be the elements of things, imagine "the whole heaven to be a musical scale and a number." He also describes and criticizes the doctrine in *De Caelo* 290b12—291a8. Plato's account may have been based on Philolaus' notion of cosmic *harmonia*(Freeman, Ancilla, 74). See Charles Kahn, "Pythagorean Philosophy Before Plato," in *The Presocratics,* ed. A.P.D. Mourelatos(Garden City, N.Y.: Anchor Press, 1974), 177. The Pythagorean formulation of the doctrine may have been stimulated by Anaximander's "wheels"(Burnet, 110) or by analogy with the seven-string lyre(Philip, *Pythagoras,* 126, 133). For a history of the music of the spheres see James Haar, "Musica Mundana: Variations on a Pythagorean Theme," (unpublished Ph.D. dissertation, Harvard University, 1960).

> The Pythagoreans, because they saw many of the attributes of numbers belonging to sensible bodies assumed existing things to be numbers...Their reason was that numerical properties are inherent in the musical scale, in the heavens, and in many other things.[52]

Philolaus too had earlier proclaimed that "you may see the nature of Number and its power at work...in all human activities and words everywhere, both throughout all technical production and also in music.[53]

The Pythagoreans were designated in antiquity as music theorists (*harmonikoi*) as distinct from performers or music teachers.[54] Plato's sole mention of them by name in any of his existing works notes they regarded harmonics and astronomy as sister sciences and that they wasted their time "measuring audible concords and sounds against one another."[55] To Philolaus we owe the terminology (if not the correct calculation) for the finer divisions of the tone —apotome, diesis, diaschisma, comma, schisma—as well as the diatonic division of the tetrachord into two 9 : 8 tones and a 256 : 243 semitone or leimma.[56] Archytas of Metapontum, a contemporary

52) *Metaphysics* N1090a20.

53) Freeman, *Ancilla*, 75.

54) Philip, *Pythagoras*, 130. On the term *harmonikoi* as Pythagorean music theorists see Mathiesen, "Problems in Terminology," 4—5 and Andrew Barker, "Music and Mathematics: Theophrastus Against the Number-Theorists," *Proceedings of the Cambridge Philological Society* 203(1977), 2, 14.

55) *Rupublic* 530c—531c. Barbera, in the article cited in note 1, challenges the traditional interpretation of this passage in which the Pythagoreans are identified as those who waste their time measuring audible concords, although he is able to suggest no other likely alternative.

56) Philolaus' divisions of the tone are transmitted by Boethius(Bower trans., 182—185) and based on information borrowed from Nicomachus' lost *Eisagoge musica*(Bower, "Boethius and Nicomachus," 10—11). Macrobius (*Commentary*, 189) states that diesis was the early Pythagorean name

of Plato and the pre-eminent Pythagorean mathematician, politician, and music theorist, contributed the interval calculations for the chromatic and enharmonic genera, the term "harmonic mean," accustical discoveries(see below), and the diatonic division of the tetrachord into a 9 : 8 tone, and 8 : 7 tone, and a 28 : 27 semitone.[57] The work of Philolaus and Archytas followed by Euclid's *Sectio canonis*(c. 300 B.C.),[58] our first preserved account of the complete system of Pythagorean harmonics, represent the culmination of what must have been a long tradition of musical-mathematical theory characterized by the manipulation of whole-number ratios.

In addition, the Pythagoreans cultivated the practice of music for its theraputic value. Aristotle's pupil, Aristoxenus, the acclaimed musician who knew "the last of the Pythagoreans," testified that they employed music to purge the soul as they used medicine to

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for leimma, but this terminology may have originated with Philolaus hismelf since leimma seems to have been the more common Pythagorean word(Levin, "Nicomachus of Gerasa," 167). The authority for Philolaus' diatonic division of the tetrachord is his Fragment 6 (Freeman, *Ancilla*, 74), but this division may not have appeared until Plato's *Timaeus*(Levin, *ibid.*, 175—183).

57) Archytas' tetrachordal divisions are preserved in Ptolemy's *Harmonica* (quoted in Levin, "Nicomachus of Gerasa," 166) and Boethius' *De Musica* (Bower trans., 323—325), while Iamblichus' *In Nicomachi arithmeticam introductionem* is the source for Archytas'(and Hippasus') naming of the harmonic mean. Nicomachus, however, gives credit to Philolaus for discovery of the harmonic mean in his *Introduction to Arithmetic*. See Richard Crocker, "Pythagorean Mathematics and Music Ⅱ," *Journal of Aesthetics and Art Criticism* 22(1964), 325—329. For Archytas' biography and scientific discoveries see Burkert, *Lore and Science*, 384—389, 442—447 and Charles A. Barbera, "The Persistence of Pythagorean Mathematics in Ancient Musical Thought," (unpublished Ph.D. dissertation, University of North Carolina at Chapel Hill, 1980), 23—29. For Archytas' political activity see Minar, *Pythagorean Politics*, 86—94. See also note 1.

58) See Thomas Mathiesen, "Euclid's Division of the Monochord." *Journal of Music Theory* 19(1975), 236—258. For the *Sectio canonis* as a Pythagorean document see Edward Lippman, *Musical Thought in Ancient Greece*(New York: Columbia University, 1964), 153—156 and Barbera, "The Persistence of Pythagorean Mathematics," 112—113, 151—161.

purge the body.[59] Implied, of course, is the well-known doctrine of ethos, hinted at in the Philolaic fragments,[60] articulated by Damon of Athens,[61] and fully elaborated by Plato in the *Republic*.[62] The Middle Ages and Renaissance knew the connection between the power of music and Pythagoreanism through the popular story of the Tauromenian youth as found in Iamblichus and Boethius:

> Among the deeds of Pythagoras likewise, it is said, that once through the spondaic song of a piper, he extinguished the rage of a Tauromenian lad, who had been feasting by night, and intended to burn the vestibule of his mistress, in consequence of seeing her coming from the house of his rival. For the lad was inflamed and excited to this rash attempt by a Phrygian song; which however Pythagoras most rapidly suppressed. But Pythagoras, as he was astronomizing, happened to meet with the Phrygian piper at an unseasonable time of night, and persuaded him to change his Phrygian for a spondaic song; through which the fury of the lad being immediately repressed, he returned home in an orderly manner, though a little before this, he could not be in the least restrained, nor would in short, bear any admonition; and even stupidly insulted Pythagoras when he met him.[63]

59) Burnet, 97. On music therapy in the Pythagorean Society see Porphyry, Hadas and Smith trans., 117 and Iamblichus, Taylor trans., 31—32, 88, 115. See also the commentary by De Vogel, 162—166.

60) Anderson, 37. See also note 70.

61) For the fragments of Damon see Freeman, *Ancilla,* 70—71. The most extensive fragment is transmitted by Aristides Quintilianus, *On Music,* trans. Thomas Mathiesen(New Haven: Yale University, 1983), 145. See also Carnes Lord, "On Damon and Music Eductaion," *Hermes* 106(1978), 32—43 and Anderson, 38—42.

62) 399e10—11. Also quoted in Strunk, *Source Readings,* 4—7.

63) Iamblichus, Taylor trans., 60. See also Boethius(Bower trans., 39—40), who cites a lost work of Cicero for his source. Abbreviated versions of the story are found in Sextus Empiricus vi. 8 and Athenaeus iv. 184e. Martianus Capella mentions Damon instead of Pythagoras(*The Marriage*

The corroborative, albeit inconclusive, testimony of Philolaus and Aristotle along with the overwhelming evidence of the Pythagoreans as music theorists and the apparent importance of music in their societal life have thus led most scholars to conclude that the discovery of musical ratios provided the stimulus for their vision of the world as number of pure form.[64]

In addition to the invention of music theory and music therapy, Pythagoras was credited with many other "firsts" by his biographers in late antiquity. According to Porphyry, Pythagoras was the first to introduce the doctrines of reincarnation, eternal recurrence, and immortality of the soul into Greece;[65] Diogenes Laertius makes

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of Philology and Mercury, trans. W.H. Stahl and R. Johnson [New York: Columbia University, 1977], 358). See also the discussion by Edmiston, 180—181. The ancient Sicilian city of Tauromenium, home of the historian Timaeus, was not founded until after the time of Pythagoras(De Vogel, 165).

64) E.g., Burnet, 107—108; Cornford, *Plato and Parmenides*, 2; Kirk and Raven, 229; Guthrie, *History*, 221. Aristoxenus suggested that Pythagoras' interest in number stemmed from its commercial applications(Guthrie, *ibid.*). Another theory is that Pythagoras, schooled in his father's trade of gem engraving, was led to the concept of the world as number by observing the perfect geometrical shapes of natural crystals. See Willian Ridgeway, "What Led Pythagoras to the Doctrine That the World Was Built of Numbers?" *Classical Review* 10(1896), 92—95.

65) Hadas and Smith trans., 113. Reincarnation or metempsychosis as a teaching of Pythagoras is supported by the ancient testimony of his contemporary, Xenophanes of Colophon(Freeman, *Ancilla*, 21—22). The origin of reincarnation along with its related notions of the kinship of all life and the immortality of the soul are obscure but could be Thracian, Indian, or even Greek, although probably not Egyptian as Herodotus(ii. 123) reports. There is no good evidence that these were Orphic beliefs taken over by the Pythagoreans as some(e.g., Minar, *Pythagorean Politics*, 125) contend. See E.R. Dodds, *The Greeks and Irrational*(Berkeley: University of California, 1951), 143—155; Jonathan Barnes, "Pythagoras and the Soul," in *The Presocratic Philosophers* I (London: Routledge & Kegan Paul, 1979), 100—120; and I. Gobry, "La doctrine pythagoricienne de l'ame," *Diotima* 7(1979), 81—85. On the doctrine of eternal recurrence see Milič Čapek, "The Theory of Eternal Recurrence in Modern Philosophy of Science," *Journal of Philosophy* 57(1960), 289—296 and Philip, *Pythagoras*, 75.

him responsible for several scientific and mathematical discoveries ;[66] and Nicomachus ascribes to him the addition of the eighth string to the lyre and the division of the tetrachord into the diatonic, chromatic, and enharmonic genera.[67] Most of these attributions, including the invention of the term "philosophy" itself,[68] can no longer be maintained without severe qualification and a few cannot possibly be true, but they represent an early and well established tradition of Pythagoras as inventor and innovator. As for his discovery of the musical ratios, however, some additio-

66) E.g., the identification of the Morning and Evening Star as the same(viii. 14), the sphericity of the earth(viii. 48), the perfection of geometry(viii. 11), and the "Pythagorean Theorem"(viii. 12). According to Iamblichus (Taylor trans., 48, 84—85) Pythagoras learned geometry in Egypt and gave it the special name, *historia*("inquiry"). Authors in late antiquity attribute several geometrical theorems and mathematical achievements to Pythagoras, but modern authorities are generally sceptical about such claims. See W.A. Heidel, "The Pythagoreans and Greek Mathematics," *American Journal of Philosophy* 61(1940), 1—33. The "Pythagorean Theorem" exists on a clay tablet dating from the time of Hammurabi. See B.L. van der Waerden, *Science Awakening*, trans. A. Dresden(New York: John Wiley, 1963), 76—77. The Theorem is related to the discovery of irrational numbers made in probably the mid-to late-fifth century B.C. See Kurt von Fritz, "The Discovery of Incommensurability by Hippasus of Metapontum," *Annals of Mathematics* 46(1945), 242—264. Crockor (p. 320) speculates that the study of irrationals began with the calculation of the geometric mean in music while Levin("Nicomachus of Gerasa," 169—171) suggests the stimulus may have been Archytas' proof that superparticular ratios cannot be halved(Boethius, Bower trans., 193—196).

67) Levin, "Nicomachus of Gerasa," 26, 33, 132—135. See also Ps.-Plutarch, *De Musica* 37. Boethius(Bower trans., 75) credits the addition of the eighth string to the Pythagorean, Lycaon of Samos. The tetrachordal divisions are more commonly attributed to Archytas(see note 57).

68) Cicero(*Tusculan Disputations* iii. 8—10), on the authority of Dicaearchus, relates the familiar parable of the "three lives." With Pythagoras, *philosophia*, which was previously equated with "curiosity," came to mean a way of life and a method of purification by which one escaped the cycle of reincarnations(Burnet, 83). See Anton-Hermann Chroust, "Some Reflections on the Origin of the Term 'Philosopher,'" *The New Scholastician* 38(1964), 423—434. The technical meaning of philosophy as an "insatiable striving for wisdom" was not established until Plato(Burkert, *Lore and Science*, 65—66). On the neologisms of Pythagoras see Vogel, 218—220.

nal material needs yet to be considered.

Like the so-called Pythagorean Theorem, the musical ratios were known in the Middle East long before the sixth century B.C.[69] The ancient Mesopotamians related them to the seasons—autumn=4 : 3, winter=3 : 2, summer=2 : 1, spring=1 : 1—displaying by these ralationships a rudimentary concept of ethos.[70] The ratios may have been attractive to Pythagoras for reasons of number magic, for which there is a wealth of testimony in the *acusmata* or symbolic sayings.[71] The four different numbers(tetrad) of which the ratios are formed add up to ten(decad), "thought to be perfect and to comprise the whole nature of number," according to Aristotle.[72] The tetractys of the decad, represented graphically as dots arranged in the figure of an equilateral triangle, was thus a sacred symbol for the Pythagoreans by which oaths were sworn(see Figure 3).[73]

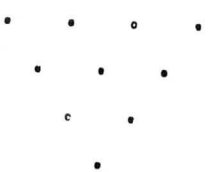

Figure 3. The Tetractys of the Decad.

69) Bowen, 86.

70) Henry George Farmer, "The Music of Ancient Mesopotamia," *New Oxford History of Music*(London: Oxford University), 253.

71) Burkert, *Lore and Science*, 166-192, 283—284. For lists and categories of *acusmata* see Iamblichus, Taylor trans., 43—47 and Diogenes Laertius viii. 17—18. The term *symbola* is synonymous with *acusmata* and stems from Aristotle. See Philip, "Aristotle's Monograph," 192—194. Most of the *acusmata* are primitive vetoes and taboos from folk religion, although Pythagoras may have added the allegorical meanings.

72) *Metaphysics* 986a8.

73) Levin, *Harmonics of Nicomachus*, 65—66. Philip(*Pythagoras*, 97—98) believes, however, that the tetractys and its supernatural powers were a post-Aristotelian development.

Another attraction of the musical ratios for the Pythagoreans may have been their superparticular construction. Such ratios had a special mathematical name("epimore") while others could only be expressed by more awkward terminology. They had long had a special function in the Middle East—the calculation of monetary interest—and must have been familiar to anyone from a highly commercialized culture like Ionia or Samos, especially to the son of a successful gem engraver. Individually such ratios could also be seen as representing the union of Odd and Even, so symbolic for Pythagorean cosmogony discussed above.[74]

Alternatively, if Pythagoras was ignorant of the musical ratios from outside sources, it has been assumed that he must have discovered them by comparing string lengths. The natural device on which to do so would have been the monochord, but date of its invention is uncertain.[75] String length proportions are not obvious on the Greek lyre, whose strings were tuned by tension, but could perhaps have been perceived on the Egyptian lyre or trigonon(with which Pythagoras would have been familiar from one of his well-attested travels), whose strings were tuned by length.[76]

Reports of the earliest acoustical experiments, though, were not necessarily of those by Pythagoreans:

> Hippasus prepared four bronze discs in such a way that their diameters were equal, while the thickness of one was 4 : 3 that

74) Burkert, *Lore and Science*, 438—442.

75) The monochord(*kanon*) is associated with Pythagoras by Aristides Quintilianus(Mathiesen trans., 161—162) and Diogenes Laertius(viii. 12). For speculations on the date of its origin see B.L. van der Waerden, "Die Harmonielehre der Pythagoreer," *Hermes* 78(1943), 170, 177.

76) Oppermann, 286—287. For a discussion of Greek stringed instruments associated with Pythagoras and their ancient references see Levin, "Nicomachus of Gerasa," 120—130.

of the second, 3 ; 2 that of the third, and double that of the fourth; when struck they made concordant intervals.[77]

Theon of Smyrna(second century A.D.) associated Hippasus, an apostate Pythagorean,[78] with the non-Pythagorean, Lasus of Hermione(both sixth century B.C.) in conducting further experiments by striking four vessels—one empty and the others filled by 1/4, 1/3, and 1/2—and subdiving strings. The struck vessels, of course, do not yield the proper results as noted adove, but would do so if used as resonance chambers.[79]

Interest in acoustics was therefore not exclusive to the Pythagoreans any more than were their theories of the nature of sound and its propagation as found in the Archytas fragments and the prologue to the *Sectio canonis*. Sound, according to both documents, results from motion or pulsation. More frequent or "swift" motions produce high notes; intermittant or slower motions produce low notes. Archytas incorrectly concluded from these observations that pitch is directly related to the force producing the motion and that higher pitches must reach our ear before lower ones. Sound, he contended, speeds to its destination on a "missile of air" much like a hurled projectile, e.g., the harder something is thorwn the faster(and farther) it travels. His acoustical experiments with sounding pipes betray this misapprehension, but, more importantly

77) The quote(Burkert, *Lore and Science*, 337) is attributed to Aristoxenus.

78) Hippasus was said to have been drowned for divulging the secret of constructing the dodecahedron. He is the oldest known Pythagorean to be active in both mathematics and music. His name is connected with both the *acusmatici* and *mathematici* by Iamblichus(Burkert, *Lore and Science*, 193—194) and he is associated with Archytas in naming the harmonic mean(see note 57). The discovery of incommensurables has also been attributed to him(see note 66).

79) See the quotation and discussion in Burkert, *Lore and Science*, 377—378 and Levin, "Nicomachus of Gerasa," 150.

for our purposes, they provide the first explicit link between such experiments and Pythagoreanism:

> Again, with *auloi* as well, when the breath issuing from the mouth falls on the openings near the mouth, it effects a higher-pitched sound because of its great force; but when the breath falls on the distant openings, it produces a lower-pitched sound...So too with the reed pipe. If one shuts off its lower end and blows, it will emit a low-pitched kind of sound; but if one blows into half the reed pipe or into any fraction of it whatsoever, it will make a highpitched sound. For the same breath passes weakly through the long passage, but forcefully through the lesser...It is clear to us from many examples that high-pitched sounds move more quickly and lower-pitched sounds more slowly.[80]

What, finally, does music have to do with metalworking? Hippasus' experiment with bronze discs suggests a relationship[81] while a Pythagorean *acusma* further states that "the ring of bronze when it is struck is the voice of a daemon entrapped in it."[82] In Phrygian myth the Idaen Dactyls, black-smiths and servants of Rhea Kybele, mother of Zeus, invented music through the rhythm of their hammers and the sounds of their anvils.[83] The story of this

80) Translation by Bowen, 82—83, whose article is a concise survey of early Greek acoustical science. The inverse relationship between length and frequency is also noted in Ps.-Aristotle, *Problems* xix. 23. 35. 50., but in Ps.-Aristotle, "De Audibilibus," 80411-8, high and low sounds are said to reach the ear simultaneously.

81) See note 77. J.A. Raasted("A Neglected Version of the Anecdote about Pythagoras's Hammer Experiment," *Cahiers de l'Institut du moyen âge grec et latin* 31a [1979], 1—9) argues convincingly that Pythagoras' hammers were transformations of Hippasus' discs.

82) Porphyry, Hadas and Smith trans., 120.

83) Burkert, *Lore and Science*, 376. Ps.-Plutarch, *De Musica* 5 connects the Dactyls with the introduction of the aulos. For the Dactyls in Greek

invention may extend back to the ancient Hittites of the thirteenth century B.C., who included Kybele as Kumbaba in their pantheon.[84] Porphyry has Pythagoras visit the Dactyls on Mt. Ida on his way to Croton:

> Coming to Crete, he visited the initiates of Morgos, one of the Idean Dactyls, by whom he was also purified with a meteorite, a ritual in which he lay prone beside the sea at dawn, and at night, wreathed with the wool of a black ram, lay beside the river. Going down with the black wool into the so-called 'Idean cave,' he spent the prescribed thrice nine days there and made offerings to Zeus...[85]

In conclusion, what the Middle ages and Renaissance knew of Pythagoras was based on ancient tradition. Because of his early reputation as a genius and demigod and his Society's preoccupation with number magic and music theory, the myth of the discovery of the musical ratios became attached to him no later than the fourth century B.C., triumphing over the reputations and activities of contemporary theorists who may have been for a time his rivals for the honor. Sometime between the fourth century B.C. and the second century A.D., Near-Eastern myths of the origin of music also became part of the Pythagorean legend. Nicomachus was probably personally responsible for conflating the disparate traditions into a colorful fairy tale destined to be repeated as fact for nearly sixteen centuries thereafter.

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mythology, see Robert Graves, *The Greek Myths* I, rev. ed.(London: Penguin Books, 1960), 185—188.

84) Eric Werner, *The Sacred Bridge*(New York: Columbia University, 1959), 376—377 and Walter Burkert, *Structure and History in Greek Mythology and Ritual*(Berkeley: University of California, 1979), 102—105, 120.

85) Hadas and Smith trans., 112—113. See also Diogenes Laertius vii. 4.

Pythagoras may not have discovered the musical ratios but he raised them to a new level of importance. Circumstantial evidence shows that the superparticular proportions held a significance for the Society far beyond practical use. Contemplation of this significance led to the concept of numbers as things or the mathematization of the physical world, one of the most seminal concepts in the history of thought.

<요 약>

피타고라스의 무지쿠스

Gene H. Anderson

음향학의 시조라 불리우는 피타고라스(Pythagoras)가 협화음일때 두 음정의 음악적 비율(musical ratio)을 발견하게 된 전설과 여기에 첨가된 내용들을 추적하여 그 근원을 밝혀 보려 시도하였다.

그 결과 중세와 르네쌍스 시대의 학자들이 피타고라스가 고안한 것으로 생각하고 있던 많은 것들이 희랍시대의 전통에 기초한 것으로 피타고라스가 음악적 비율을 발견하였다는 전설도 사실이 아닐 수 있다는 가능성을 제시하였다.

(김 문 자)